CSx25: Digital Signal Processing
NCS224: Signals and Systems

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Outline

- ~~Digital Signal Processing Introduction~~
  - ~~Mathematical modeling~~
  - ~~Continuous Time Signals~~
  - ~~Discrete Time Signals~~
- ~~Analyzing Continuous-Time Systems in the Time Domain~~
- ~~Analyzing Discrete Systems in the Time Domain~~
- ~~Fourier Analysis for Continuous-Time Signals and Systems~~
- ~~Sampling and Reconstruction~~
- **Analysis and Design of Filters**

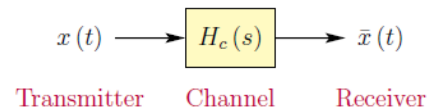
Filters

- In many signal processing applications the need arises to change the strength, or the relative significance, of various frequency components in a given signal. Sometimes we may need to *eliminate certain frequency components* in a signal; at other times we may need to *boost the strength of a range of frequencies* over others.
- This act of changing the relative amplitudes of frequency components in a signal is referred to as filtering, and the system that facilitates this is referred to as a **filter**.

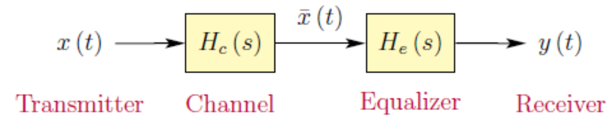
Filters

Concept of "filter"

Example: Communication channel



Example: Communication channel with an equalizer



$$H_c(s)\ H_e(s) = 1 \qquad \Rightarrow \qquad H_e(s) = \frac{1}{H_c(s)}$$

Filters

Distortionless transmission

Definition

A CTLTI system is said to be *distortionless* if its output signal is simply a scaled and delayed version of its input signal.

$$y(t) = K\,x(t - t_d)$$

K and t_d are constants.

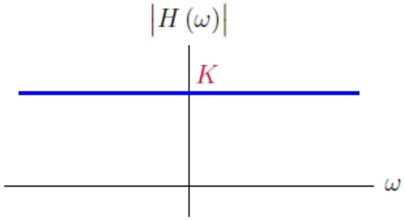
$$Y(\omega) = K\,e^{-j\omega t_d}\,X(\omega)$$

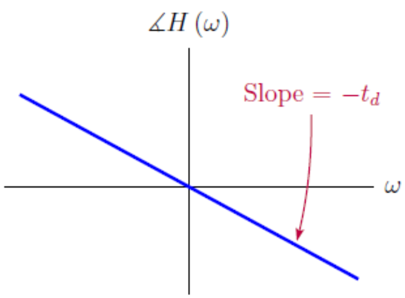
System function needed for distortionless transmission:

$$H(\omega) = K\,e^{-j\omega t_d}$$

$$|H(\omega)| = K$$

$$\Theta(\omega) = \measuredangle H(\omega) = -\omega t_d$$

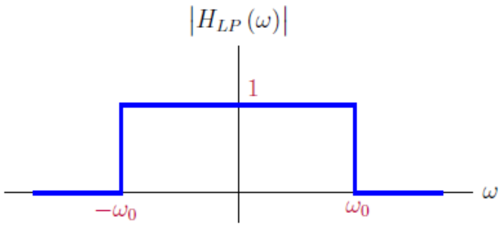




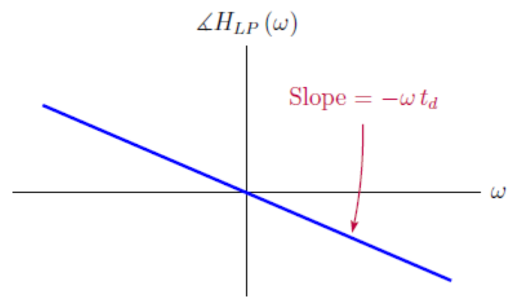
Filters

Ideal filters

Ideal lowpass filter: $H_{LP}(\omega) = \Pi\left(\frac{\omega}{2\omega_0}\right) e^{-j\omega t_d} = \begin{cases} e^{-j\omega t_d}\ , & |\omega| \leq \omega_0 \\ 0\ , & |\omega| > \omega_0 \end{cases}$

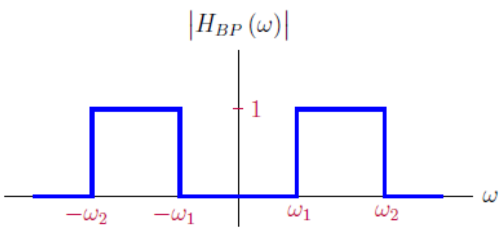


(a)

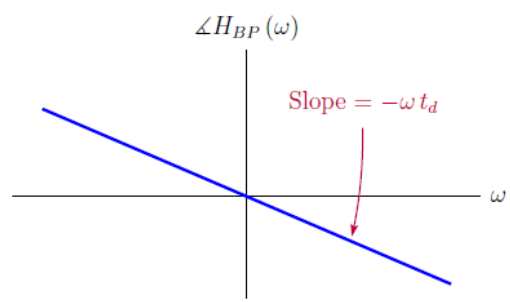


(b)

Ideal bandpass filter: $H_{BP}(\omega) = \begin{cases} e^{-j\omega t_d}\ , & \omega_1 \leq |\omega| \leq \omega_2 \\ 0\ , & \text{otherwise} \end{cases}$



(a)

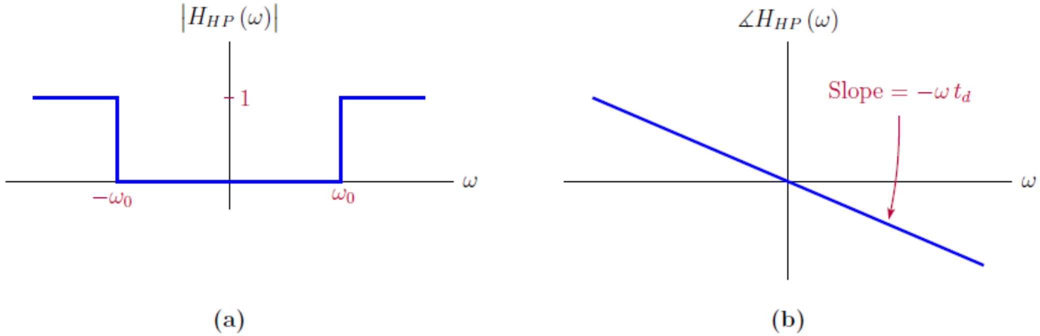


(b)

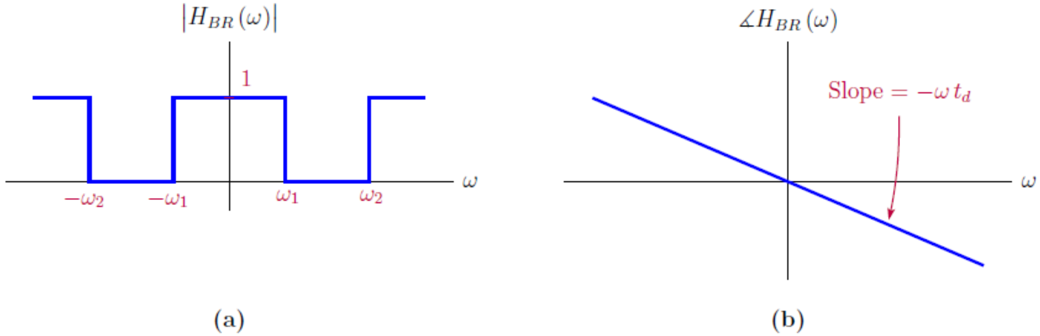
Filters

Ideal filters (continued)

Ideal highpass filter



Ideal band-reject filter



Filters

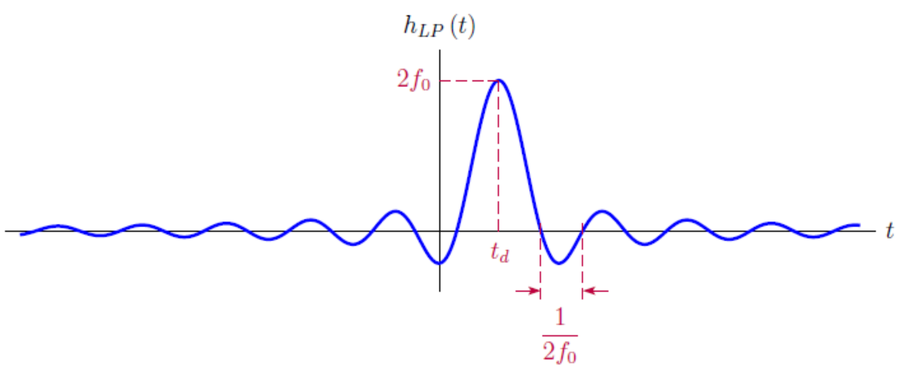
Ideal filters (continued)

Impulse response of ideal lowpass filter:

$$h_{LP}(t) = \mathcal{F}^{-1}\left\{H_{LP}(\omega)\right\} = \frac{\omega_0}{\pi}\,\mathrm{sinc}\left(\frac{\omega_0}{\pi}(t - t_d)\right)$$

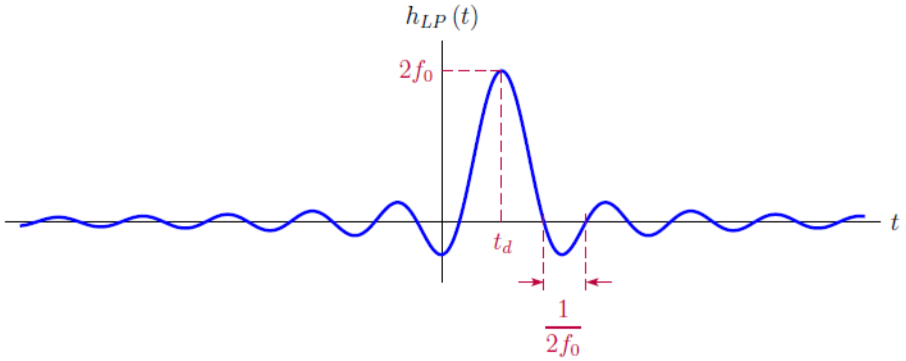
Using f_0 instead of ω_0:

$$h_{LP}(t) = 2f_0\,\mathrm{sinc}\left(2f_0(t - t_d)\right)$$



Filters

Ideal filters (continued)



Observations regarding $h_{LP}(t)$:

1. The impulse response $h_{LP}(t)$ is in the form of a sinc function with a peak amplitude of $2f_0$. This peak occurs at the time instant $t = t_d$.
2. The zero-crossings of the impulse response are uniformly spaced. Consecutive zero-crossings are $1/(2f_0)$ apart from each other.
3. The impulse response exists for all t including $t < 0$. Ideal lowpass filter is a non-causal system, and is therefore physically unrealizable.
4. No amount of additional time delay would make the impulse response causal.

Filters

Interactive demo: ilpf_demo.m

Experiment by changing parameters f_0 and t_d.